# The Impact of Deterministic and Nondeterministic Components on ePRTC Holdover

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- The enhanced primary reference time clock (ePRTC) requires a phase drift of no more than 100 ns after 14 days during the absence of an external time reference to UTC.
- The phase error $x(\tau_h)$ in the clock equation during holdover is stochastic, composed of deterministic and nondeterministic functions.
- The poster separates the impact due to deterministic temperature variations from the nondeterministic components to improve prediction capabilities.

## Clock Equation

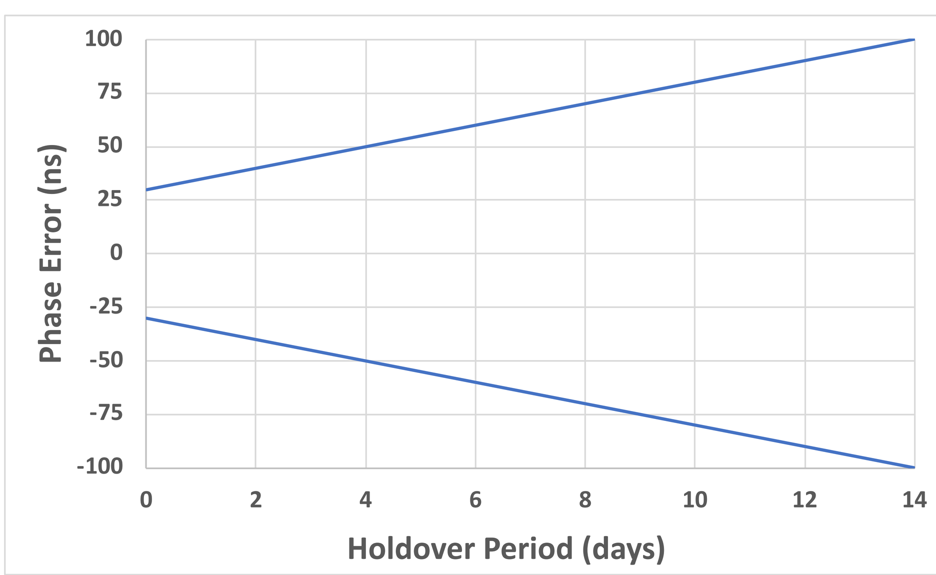
$$x(\tau_h) = \frac{D\tau_h^{\ 2}}{2} + \int_0^{\tau_h} E(T,P,M,H,G,\ldots)dt + y_0\tau_h + x_o + \varepsilon(\tau_h)$$

- $\tau_h$: holdover period
- $x(\tau_h)$: phase error for holdover period
- D: linear aging term

- E : frequency change due environmental effects:
  - T: temperature
  - P: pressure
  - M: magnetism
  - H: humidity
  - G: acceleration

- $y_o$ : starting syntonization error (initial frequency offset)
- $x_o$ : starting synchronization error (initial phase offset)
- $\varepsilon(\tau_h)$: cumulative wander due to system noise

## 1. ePRTC Phase Error Requirements




- ePRTC per ITU-T G.8272.1 time error after 30 days on power
- $|x(\tau_h)| \leq 30 + 5.787037\text{E-}5\ \tau_h$

## 2. Cesium and the Clock Equation

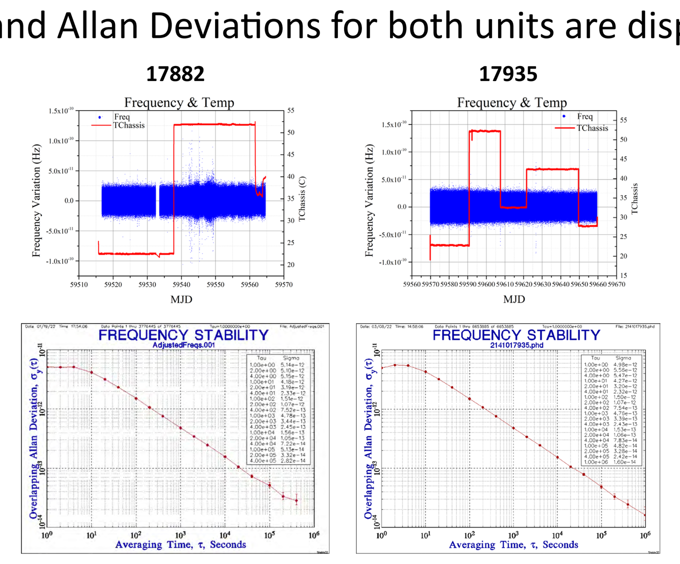
- ePRTC systems primarily employ cesium references as the local clock
- For a cesium reference:
  - D= 0
  - $y_o$ - determined by learning period, source noise, external reference quality and quality of learning. This poster will focus on source noise at the end of the learning period as the dominant factor.
  - $x_o$ - determined by source noise, external reference quality and quality of learning – typically < 1ns.
  - $\varepsilon(\tau_h)$- determined by cesium reference noise.
  - E - for most ePRTC deployments the cesium is placed in a well-controlled stationary environment. Cesium references are relatively immune to the small changes that may occur in these environments.
    In this poster temperature is assumed to be the onlyenvironmental factor that will have a measurable impact upon holdover.

## 3. Cesium Characterization and Holdover Modeling

- Due to the superior accuracy and precision of cesium frequency references, statistically meaningful measurements of some of the characteristics that affect the phase error during holdover take weeks to months to measure and are impractical to test in production.
- A multi year study of 10 cesium references with a minimum of 40 days testing each is being conducted to characterize the noise and temperature effects on production cesium references.
- Prior to the WSTS 2022 Workshop, data for 2 of the 10 units has completed.
- A model based on the clock equation is developed for both deterministic and nondeterministic components.
- The nondeterministic component model for one of the units is compared to the holdover estimator presented in “Optimizing Resilient Time Delivery Through Estimation Calculations” on day of the WSTS 2022 workshop.

## 4. Test Methodology

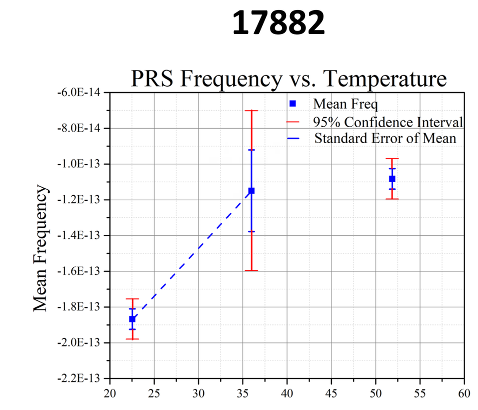
- Units placed in temperature chambers one at az time
- Phase and frequency measured for at least 14 days at two or more temperatures.
- Actual test times and number of temperatures varied based on unit availability and results of prior runs.
- Units compared to hydrogen maser steered by TWSTT.
- Frequency and Allan Deviations for both units are displayed below.




## 5. Temperature Coefficients

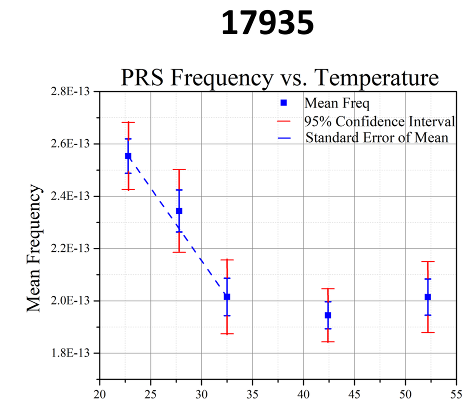
- The temperature coefficients were small relative to the frequency noise of units as witnessed in the frequency charts.
- Both units displayed temperature coefficients $m_T$ of similar magnitude in the region of 20 to 30 °C, and then flattened out above this range.

17882




$m_T$=5.14 E-15

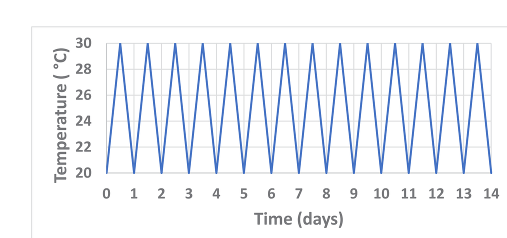
17935



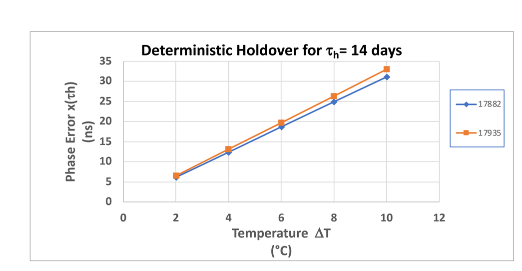

$m_T$=5.45 E-15

## 6. Incorporating the Deterministic Function E(T) within the Clock Equation

- To illustrate the deterministic impact of E(T), temperature will be assumed to be a periodic linear diurnal ramp between 20 °C to 30 °C to 20 °C.




- In this region E(T) is a linear function E= $m_T\,\Delta T$.
  In the worst case where holdover starts at 20 °C or 30 °C
  $\int_0^{\tau_h} E(T)d(\tau)$ reduces to $\frac{m_T}{2}\Delta T\tau_h$ ($\tau_h$ =1d,2d,3d…).




## 7. Nondeterministic Portions of the Clock Equation

- Two components of the clock equation can be modeled as nondeterministic.
- In an ideal loop, the best possible frequency offset distribution is the Allan Deviation at the learning period of the loop $(\tau_l)$ hence $y_o\,\tau_h = \sigma_y\,(\tau_l)\,\tau_h$.
- The cumulative wander term can be approximated as

$$\varepsilon(\tau_h) = \frac{mod\sigma_y\,(\tau_h)}{\sqrt{3}}\tau_h \sim \frac{\sigma_y\,(\tau_h)}{\sqrt{3}}\tau_h.$$

- Assuming both units reached the flicker floor at 1,000,000 seconds then for $(\tau_l)$=14 days and $(\tau_h)$ =14 days

$$\sigma_y\,(\tau_l) = \sigma_y\,(\tau_h) = \sigma_y\,(1E6s) \text{ and}$$

$$\sigma(\tau_h) = \sqrt{(\sigma_y\,(\tau_l)\tau_h)^2 + \left(\frac{mod\sigma_y\,(\tau_h)}{\sqrt{3}}\tau_h\right)^2} \sim \frac{2}{\sqrt{3}}\sigma_y\,(1E6s)\tau_h$$

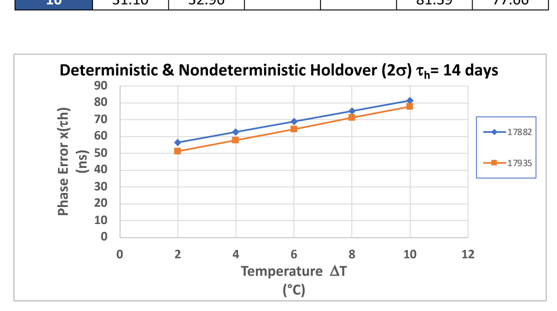
- (Unit 17882 did not have a long enough run for a valid 1E6s data point. This poster assumes it continued on the same slope until 1E6 s resulting in ~1.8E-14 )

## 8. Finalized Model

- Combining the deterministic and nondeterministic components for time periods 14 days and greater.

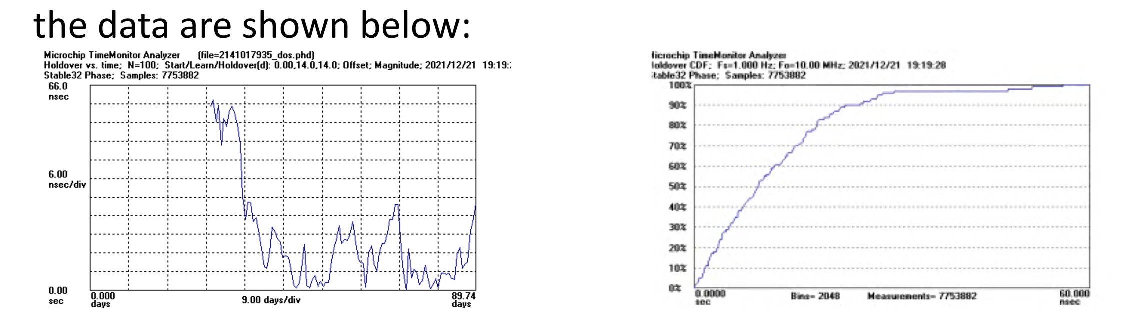
$$x(\tau_h) = \left(\frac{m_T}{2}\Delta T + \frac{2}{\sqrt{3}}\sigma_y\,(1E6s)\right)\tau_h$$
$$(\tau_h = 14d, 15d, \ldots).$$

| | Deterministic | | Nondeterministic (2σ) | | Total (2σ) | |
|---|---|---|---|---|---|---|
| ΔT (°C) | 17882 | 17935 | 17882 | 17935 | 17882 | 17935 |
| 2 | 6.22 | 6.59 | 50.28 | 44.70 | 56.50 | 51.29 |
| 4 | 12.44 | 13.18 | | | 62.72 | 57.88 |
| 6 | 18.66 | 19.78 | | | 68.94 | 64.47 |
| 8 | 24.88 | 26.37 | | | 75.17 | 71.06 |
| 10 | 31.10 | 32.96 | | | 81.39 | 77.66 |




## 9. Holdover Estimator

- The WSTS 2022 Workshop presentation “Optimizing Resilient Time Delivery Through
  Estimation Calculations” includes a holdover estimator for determining the nondeterministic distribution of future holdover data based off multiple actual vs. predicted holdover calculations within a frequency data set.
- The results of the estimator and cumulative distribution function for the data are shown below:



- There is close agreement in the 2 sigma values of both approaches with the slightly higher value in the model attributable to the $mod\sigma_y\,(\tau_h) \sim \sigma_y\,(\tau_h)$ approximation and the assumption that the flicker floor began at $\tau$=1E6s